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Orientifold, geometric transition and large N duality for SO / Sp gauge theories

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Abstract: We extend the large N duality of four dimensional $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with additional chiral fields and arbitrary superpotential recently proposed by Cachazo, Intriligator and Vafa to the case of SO / Sp gauge groups. By orientifolding the geometric transition, we investigate a large N duality between $\mathcal{N} = 1$, SO / Sp supersymmetric theories with arbitrary superpotential and an abelian $\mathcal{N} = 2$ theory with supersymmetry broken to $\mathcal{N} = 1$ by electric and magnetic Fayet-Iliopoulos terms.

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Contents

1. Introduction

String theories on conifolds have been used to study large N gauge theories. In [1], it was conjectured that there is a large N duality between type IIB string theory on $AdS_5 \times T^{1,1}$ and $\mathcal{N} = 1$ conformal supersymmetric theory in four dimensions with gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N)$. The dualities have been soon extended to more general conifold type singularities [2]. The renormalization group flow has been studied after fractional D-branes are introduced which corresponds to relevant deformation of the conformal field theory [3, 4]. Furthermore, it was shown [5] that the moduli space becomes non-commutative if discrete torsion is present on the conifold which gives rise to marginal deformation.

Some months ago, a new direction has been developed concerning type II strings at conifold singularities, and a large N duality has been proposed by Vafa between gauge systems with $\mathcal{N} = 1$ supersymmetry in four dimensions and superstrings propagating on non-compact Calabi-Yau manifolds with RR fluxes [6]. Via geometric transition, type-IIA theory with N D6-branes wrapped on a special lagrangian 3-cycle of a Calabi-Yau is dual to type-IIA theory on Kähler deformed Calabi-Yau where the 3-cycle has been replaced by an exceptional $\mathbb{P}^1$. On the Kähler deformed side, the D-branes disappear and are replaced by N units of RR fluxes through the exceptional $\mathbb{P}^1$. This duality proposal is based on the embedding of the Chern-Simons/topological string duality of [7] into superstring theory. From the M-theory perspective, this duality can be obtained by a geometrical flop on 7 dimensional manifolds with G_2 holonomy [8, 9]. The supergravity picture of this geometrical transition was then studied in [10].

More recently, this large N duality proposal has been extended to four dimensional $\mathcal{N}=1$ supersymmetric SU(N) Yang-Mills theory with an adjoint chiral field and arbitrary superpotential by Cachazo, Intriligator and Vafa [11]. This theory is claimed to be dual, through a similar geometric transition, to an abelian theory where supersymmetry is broken to $\mathcal{N}=1$ by means of Fayet-Iliopoulos terms. Most strikingly, the glueball chiral superfields, whose first components are the gaugino bilinears, are identified with certain compact periods in a non-compact Calabi-Yau, while the cutoff of the otherwise divergent non-compact periods gives an elegant explanation of the running of the coupling constant. Furthermore, the duality in [11] allows to compute the exact quantum effective superpotential of the low energy SU(N) theory.

In this paper, we extend the results of [11] to the case of SO / Sp gauge groups. The main idea is to obtain a large N duality for the SO / Sp gauge groups by acting by orientifolding on both sides of the duality proposed for SU(N) gauge theories. The complex conjugation provides an orientifolding on the conifold which can be extended to both Kähler and complex deformed conifolds to be considered. In the case without superpotential, the three cycles S^3 on the complex deformed conifold will be invariant under the orientifolding and in the dual picture, the fluxes will be through an RP^2 cycle instead of the $\mathbb{P}^1$ of U(N), after the Z_2 projection. The $\mathcal{N}=1$, SO / Sp super Yang-Mills theory with adjoint chiral field Φ and arbitrary tree level superpotential can be geometrically engineered by perturbing a conifold to a non-compact Calabi-Yau space which has only conifold singularities, and then orientifolding. In this process, the compatibility with the orientifolding will impose a constraint on complex and Kähler deformations of the Calabi-Yau space implying a particular arrangement of the 3-cycles and the 2-cycles along a complex line. In the Kähler deformed Calabi-Yau, an RP^2 will be located at the origin of the complex line while pairs of $\mathbb{P}^1$ cycles will be located along its imaginary axis symmetrically with respect to the origin. The orientifolding will map one $\mathbb{P}^1$ into the other in the pair. In [12], the same orientifolding process has been considered in the context of a duality between SO / Sp Chern-Simons gauge theory and topological string theory on an orientifold of the small resolution of the conifold.

Based on the orientifolded geometric transition explained above, we propose a large N duality for $\mathcal{N}=1$, SO / Sp supersymmetric theories with arbitrary superpotential and an abelian $\mathcal{N}=2$ theory with supersymmetry broken to $\mathcal{N}=1$ by electric and magnetic Fayet-Iliopoulos terms. The SO / Sp group is broken in a generic $\mathcal{N}=1$ vacuum, and the theory reduces at low energies to an abelian U$(1)^n$ supersymmetric theory with $N-n$ massless monopoles that admits a Seiberg-Witten formulation. An important check on the proposed duality is to show that the periods obtained from the reduced Seiberg-Witten theory arising in the world-volume of the D5-branes on the Kähler deformed conifold are consistent with the ones obtained by using the symplectic geometry on the complex deformed conifold. This implies that the couplings of the abelian gauge fields agree in both dual theories. To show this, we have used the results of [13] following the ideas of [11]. We show that the equations of both hyperelliptic curves of degree $4n+2$ agree up to a degree $2n$ starting from the highest degree.

The results obtained could be connected to the ones constructed via brane configurations with D4-branes, NS-branes and orientifold four or six planes where one rotates the NS-branes with respect to each other [14]. The rotation is similar to giving mass or introducing more general superpotentials for the adjoint chiral field.

The content of the paper is as follows. In section 2 we geometrically engineer the general $\mathcal{N}=1$, SO / Sp theory with an adjoint chiral field and a generic superpotential. We give a detailed discussion of the required orientifolding, and compute the effective superpotential showing that the regularization of non-compact periods give raise to the expected running of the coupling constant, with the appropriate β function, for SO / Sp gauge theories. We consider the action by orientifolding on the geometric transition of [11], and obtain the geometric realization of symmetry breaking patterns of the form $\mathrm{SO}(N) \to \mathrm{SO}(N_0) \times \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_n)$, or $\mathrm{Sp}(N) \to \mathrm{Sp}(N_0) \times \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_n)$. In section 3 we investigate strongly coupled field theory implications from the proposed large N duality for SO / Sp. In particular, we compare the reduced Seiberg-Witten curve with one obtained from the dual geometry. In passing, we give a new derivation of the exact low energy effective superpotential induced by generic microscopic deformations within the framework of the Whitham hierarchy formalism of softly broken $\mathcal{N}=2$ supersymmetric gauge theories.

2. Geometric engineering of $\mathrm{SO}(N)$ gauge theories with superpotential

We begin by explaining the geometric backgrounds we will be dealing with in this paper. The conifold is a three dimensional hypersurface singularity in C^4 defined by:

$$\mathcal{C}: \quad x^2+y^2+z^2+w^2=0\,. \tag{2.1}$$

We consider two kinds of deformations of the conifold, i.e. complex and Kähler. A complex deformed conifold given by

$$k := x^2+y^2+z^2+w^2-\mu=0 \tag{2.2}$$

will be considered. This is isomorphic to T^*S^3 as a symplectic manifold after the rotation of symplectic structure by the phase of μ. On the other hand, the Kähler deformed conifold to be considered will be a small resolution of the conifold. By this process, the singular point of the conifold $\mathcal{C}$ will be replaced by a $\mathbb{P}^1$ with normal bundle $\mathcal{O}(-1)+\mathcal{O}(-1)$.

In type-IIA string theory, a four dimensional $\mathcal{N} = 1$, U(N) supersymmetric gauge theory is obtained by wrapping N D6-branes on the S^3 in the complex deformed conifold. In [6], Vafa has proposed a duality, in the large N limit, between this theory and type-IIA superstrings without D-branes propagating on the Kähler deformed conifold. This duality emerges as the embedding of the large N Chern-Simons/topological string duality of Gopakumar and Vafa [7] in ordinary superstrings. The branes are replaced by N units of RR flux through $\mathbb{P}^1$, and also NS flux on the dual four cycle. Acting by orientifolding on the duality of [7], the large N duality of SO and Sp Chern-Simons gauge theory on S^3 and topological strings on an orientifold of the small resolution of the conifold has been studied in [12]. This last paper also discuss the embedding of this duality in ordinary superstrings. Note that the orientifolding acts on the conifold $\mathcal{C}$ via the complex conjugation

$$(x, y, z, w) \to (\bar{x}, \bar{y}, \bar{z}, \bar{w}) , \tag{2.3}$$

and can be extended both to complex and Kähler deformations considered above provided that μ is real. On the complex deformed conifold T^*S^3, the special lagrangian S^3 is invariant under this complex conjugation and hence the orientifold is a O6 plane wrapping it [12, 15]. To see the effect of the orientifolding on the Kähler deformed conifold, we introduce new coordinates

$$a = x + iy , \qquad b = z + iw , \qquad c = x - iy , \qquad d = -z + iw , \tag{2.4}$$

in terms of which the singular conifold is given by

$$ac - bd = 0 . \tag{2.5}$$

The complex conjugation (2.3) corresponds to the action

$$a \to \bar{c} , \qquad b \to -\bar{d} , \qquad c \to \bar{a} , \qquad d \to -\bar{b} . \tag{2.6}$$

The Kähler deformation considered above can be described as a union of two smooth 3-dimensional complex manifolds, namely $M_1 = \{(a', b, c, d) \in C^4 | d = a'c\}$ and $M_2 = \{(a, b', c, d) \in C^4 | c = b'd\}$ glued together by the natural identification and the extra condition $b = ab'$. The smooth manifold obtained in this way maps onto the conifold. Notice that the singular point $(0, 0, 0, 0)$ of the conifold is now replaced by $\mathbb{P}^1$ whose coordinates are given by a' on M_1 and b' on M_2. One can see that the complex conjugation sends a' to $-\bar{b}'$ which is the antipodal map on S^2. The quotient by this action will be RP^2.

The $\mathcal{N} = 1$ supersymmetric gauge theory on the world-volume of N D6-branes sitting on top of O6 planes wrapping the S^3 of the complex deformed conifold, has SO(N) or Sp(N) gauge group depending on the choice of the sign for the world-sheets with crosscaps. By the arguments of [6], this theory is equivalent to type-IIA string theory on the orientifold of the Kähler deformed conifold [12]. We may give a mirror description of this large N duality which reverses the direction of the transition. Now, in the large N limit, SO(N) or Sp(N) theories obtained from type IIB theory by wrapping N D5-branes on the $\mathbb{P}^1$ in the orientifold of the Kähler deformation of the conifold, are equivalent to type IIB theory on the corresponding orientifold of the complex deformed conifold (2.2), with N units of RR flux through S^3. There is also some NS flux through the non-compact cycle. The value of μ is fixed by the fluxes and this is captured by a superpotential for the chiral field S, whose first component is proportional to μ [6]. We may assume that μ is real after rotating it back by its phase. A rational three form

$$\Omega = \frac{dx\, dy\, dz}{\partial k/\partial w} \sim \frac{dx\, dy\, dz}{\sqrt{\mu - x^2 - y^2 - z^2}}\,, \tag{2.7}$$

will be a holomorphic form on the deformed conifold (2.2). On the complex deformed conifold, there is a single compact cycle $A \cong S^3$ and the corresponding dual non-compact cycle B. The A period of the holomorphic 3-form Ω is S. There are N units of RR flux though A, and also NS flux α through B; α is identified with the bare coupling of 4d SO / Sp gauge theory similarly to what happens for SU [6]. To compute the periods, consider a projection p from the conifold $\mathcal{C}$ to x-plane. Then the inverse image of the real interval $[-\sqrt{\mu}, \sqrt{\mu}]$ is the 3-cycle A and that of the line segment $[\sqrt{\mu}, \infty)$ will be the 3-cycle B. Thus, the A-period is given by

$$S = \int_A \Omega = \int_{[-\sqrt{\mu},\sqrt{\mu}]} \int_{p^{-1}(x)} \Omega = \frac{1}{2\pi i} \int_{-\sqrt{\mu}}^{\sqrt{\mu}} dx \sqrt{x^2 - \mu} = \frac{\mu}{4}\,, \tag{2.8}$$

as in the case of SU(N) [11]. The B period is divergent, and thus a cutoff Λ must be introduced as

$$\begin{aligned} \Pi &= \int_B^{(\Lambda)} \Omega = \int_{[\sqrt{\mu},\Lambda^{3/2}]} \int_{p^{-1}(x)} \Omega = \frac{1}{2\pi i} \int_{\sqrt{\mu}}^{\Lambda^{3/2}} dx \sqrt{x^2 - \mu} \\ &= \frac{1}{2\pi i} \left(\frac{1}{2}\Lambda^3 - 3S \log \Lambda - S(1 - \log S) \right) + O(1/\Lambda)\,. \end{aligned} \tag{2.9}$$

The effect of the fluxes in the four dimensional theory is an effective superpotential of the form [16]

$$W_{\text{eff}} = \int \Omega \wedge (H^{RR} + \tau N^{NS})\,, \tag{2.10}$$

where τ is the complexified coupling constant of type IIB strings. If we wrap N D6-branes on S^3, the effective action after orientifolding can be written as

$$W_{\text{eff}} = \left(\frac{N}{2} \mp 1 \right) \Pi - 2\pi i \alpha S\,, \tag{2.11}$$

because only half of the N units of RR flux are left by the orientifolding operation and the orientifold carry RR charge of ∓ 4 units for, respectively, SO and Sp. There is an additional $\pm\Pi$ coming from the RP^2 worldsheet of unoriented string. As in [11], inserting (2.8) and (2.9) into (2.10) and neglecting an irrelevant constant, we have

$$W_{\text{eff}} = \left(\frac{N}{2} \mp 1\right) \left(3S \log \Lambda + S(1 - \log S)\right) - 2\pi i \alpha S\,, \tag{2.12}$$

so finite results are reached by regularizing α in such a way that the following combination is a constant

$$3\left(\frac{N}{2} \mp 1\right) \log \Lambda - 2\pi i \alpha = const\,. \tag{2.13}$$

Now it is clear that, if we identify α with the bare coupling of SO or Sp gauge theories as $2\pi i\alpha = 8\pi^2/g^2(\Lambda)$, the previous equation reproduces the running of the coupling constant

$$\frac{8\pi^2}{g^2(\Lambda)} = 3\left(\frac{N}{2} \mp 1\right) \log \Lambda + const.\,, \tag{2.14}$$

with the appropriate β function, provided we also identify Λ with the scale of the theory. It is also immediate, following the ideas of [11], to show that there are $N \mp 2$ supersymmetric vacua where $S \neq 0$ for appropriate normalization of the superpotential. Following [6], S is identified with the SO / Sp glueball chiral superfield.

In Calabi-Yau string vacua, there is a hypermultiplet which becomes light at conifold singularities in the moduli space and can be excited [17]. Turning on fluxes causes supersymmetry to softly break from $\mathcal{N} = 2$ to $\mathcal{N} = 1$. The $\mathcal{N} = 2$ vector multiplet consists of an $\mathcal{N} = 1$ chiral superfield S and an $\mathcal{N} = 1$ U(1) photon. As orientifolding reverses the sign of U(1), the U(1) gauge symmetry is broken down to Z_2. This Z_2 symmetry is to be identified with the global Z_2 symmetry in the $\mathcal{N} = 1$ SO(N) gauge theory. On the deformed conifold, the gauge theory on N D6-branes wrapping S^3 is $\mathcal{N} = 1$ U(N) gauge theory. By orientifolding, the U(N) adjoint representations become O(N) representations, but as the O(N) group is disconnected, the gauge theory becomes that of SO(N) with O(N)/ SO(N) = Z_2 as a global symmetry. In the case of SO($2N$), there are particles created by the pfaffian $\mathrm{Pf} = (1/N!)\epsilon^{i_1 j_1 \ldots i_N j_N} \Phi^{i_1 j_1} \cdots \Phi^{i_N j_N}$ of the adjoint field Φ that are charged under Z_2 and then annihilate in pairs [18]. Since the pfaffian operator vanishes for SO($2N+1$) and Sp(N), we do not expect corresponding particle states in such cases.

We shall now concentrate on the case of $\mathcal{N} = 1$ SO(N) gauge theory with matter Φ in $(\frac{1}{2}N(N+1)-1)$ dimensional traceless tensor representation of SO(N), i.e. adjoint representation, with superpotential

$$W_{tree}(\Phi) = \sum_{k=1}^{n+1} \frac{g_k}{2k} \mathrm{Tr}\ \Phi^{2k}\,. \tag{2.15}$$

The geometric engineering is essentially the same as in [11]. We only need to take care of orientifolding. When $W_{tree}(\Phi) = 0$, the field theory is $\mathcal{N} = 2$ Yang-Mills theory. The geometric background will be the blow-up of the singular locus given by $y = z = w = 0$ on the hypersurface defined by

$$y^2 + z^2 + w^2 = 0\,. \tag{2.16}$$

Then we have a one dimensional family of $\mathbb{P}^1$'s along x-axis. Each $\mathbb{P}^1$ has normal bundle $\mathcal{O}(-2)+\mathcal{O}(0)$. From the geometry, it is clear that we can identify the blow-up of the above hypersurface with the total space of the normal bundle of $\mathbb{P}^1$ and x-axis can be identified with $\mathcal{O}(0)$ direction. If we wrap N D5-branes on a $\mathbb{P}^1$ in the family, we obtain an $\mathcal{N} = 2$ U(N) gauge theory on their uncompactified world-volume. The one dimensional moduli of $\mathbb{P}^1$ corresponds to the moduli of the Coulomb branch in $\mathcal{N} = 2$ gauge theory through identification of the position of each brane with an eigenvalue of the adjoint chiral field Φ. On this geometric background, we consider the orientifolding on $\mathcal{O}(-2) + \mathcal{O}(0)$ over $\mathbb{P}^1$ induced by the complex conjugation

$$x \to \bar{x}\,, \qquad y \to \bar{y}\,, \qquad z \to \bar{z}\,, \qquad w \to \bar{w}\,, \tag{2.17}$$

on the ambient space C^4. Because of the orientifolding we are considering, the freedom to move the D5-branes is constrained. Since the eigenvalues of SO(N) adjoint field appear in pairs $ia, -ia$ (where a is a real number), the D5-branes can be moved only in pairs along the imaginary x-axis. Here we have chosen the complex form of SO adjoint so that the eigenvalues will be purely imaginary numbers which is consistent with our choice of the orientifolding. The pair is in the reflection of each other under the orientifolding. In order to make perturbation by the superpotential (2.15), we need to freeze the $\mathbb{P}^1$'s with the normal bundle $\mathcal{O}(-2) + \mathcal{O}(0)$ at the particular locations of x given by the zeros of

$$W'(x) = g_{n+1} x \prod_{j=1}^{n} (x^2 + a_j^2)\,, \quad a_j > 0\,. \tag{2.18}$$

To explain the geometry of $\mathcal{O}(-2)+\mathcal{O}(0)$ over $\mathbb{P}^1$, we introduce two copies of C^3 with coordinates z, x, u (resp. z', x', u') for the first (resp. second) C^3. Then $\mathcal{O}(-2)+\mathcal{O}(0)$ over $\mathbb{P}^1$ is obtained by gluing two copies of C^3 with the identification:

$$z' = \frac{1}{z}\,, \qquad x = x'\,, \qquad u' = uz^2\,. \tag{2.19}$$

The z (resp. z') is a coordinate of $\mathbb{P}^1$ in the first (resp. second) C^3. Similarly, x, x' (resp. u, u') are coordinates of $\mathcal{O}(0)$ (resp. $\mathcal{O}(-2)$) direction. Now we perturb this geometry by the following change in (2.19):

$$z' = \frac{1}{z}\,, \qquad x = x'\,, \qquad u' = uz^2 + W'(x) z\,, \tag{2.20}$$

and in this way we now obtain $\mathbb{P}^1$'s only where $W'(x) = 0$, i.e. $x = 0$ and $x = \pm i a_j$, whose normal bundle is $\mathcal{O}(-2) + \mathcal{O}(0)$. Unlike the family of $\mathbb{P}^1$'s before the perturbation, we cannot move these $\mathbb{P}^1$'s. Under the orientifolding, the $\mathbb{P}^1$ located at $x = 0$ becomes RP^2 and the $\mathbb{P}^1$ located at ia_j will map to the $\mathbb{P}^1$ located at $-ia_j$. From now on, we will assume that a_j's are distinct and nonzero. We will consider the cases of SO($2N$) and SO($2N+1$), the Sp(N) case following straightforwardly. We can distribute the $2N$ (resp. $2N+1$) D5-branes among the vacua $x = 0$ and $x = \pm ia_j$ by wrapping $2N_0$ (resp. $2N_0+1$) branes around RP^2 at $x = 0$ and N_j branes around $\mathbb{P}^1$ at $\pm ia_j$, where $\sum_{j=0}^{n} N_j = N$. For the former, RP^2 is stuck on the orientifold and it gives $\mathcal{N} = 1$, O($2N_0$) (resp. O($2N_0 + 1$)) gauge theory on the world-volume. Otherwise, the orientifold projection identifies the states at ia_j with those of $-ia_j$ and we instead obtain U(N_j) gauge theory. So this is a geometric realization of the breaking

$$\mathrm{O}(N) \to \mathrm{O}(N_0) \times \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_n)\,. \tag{2.21}$$

To present the large N duality, we blow-down these $\mathbb{P}^1$'s to a singular hypersurface in C^4 which has been described in [11]. After changing variables, the blown-down Calabi-Yau is given by

$$W'(x)^2 + y^2 + z^2 + w^2 = 0\,. \tag{2.22}$$

In our case, $W'(x)$ having distinct roots, we have $2n+1$ distinct conifold singularities. If we take a small resolution at every singular point, then we obtain a set of $\mathbb{P}^1$'s whose normal bundle is $\mathcal{O}(-1) + \mathcal{O}(-1)$. These $\mathbb{P}^1$'s are called exceptional. Under the orientifolding, the exceptional $\mathbb{P}^1$'s behave the same way as before.

Following the large N duality proposal of [11], we consider a complex deformation of (2.22). Topologically, this process creates $n+1$ finite size S^3's which can be thought of as been shrunken to zero size at the singularities of (2.22). At each conifold point of (2.22), there is a one dimensional complex deformation space parametrized by a complex number μ_i. As we have $2n + 1$ conifold singular points for a generic $W'(x)$, the complex deformation space will be $2n+1$-dimensional. Being constrained by the orientifolding, the actual deformation space is $n+1$ dimensional and parametrized by $\mu_0, \ldots, \mu_n$. This can be achieved by a polynomial $f(x)$ of degree $2n$ in x with values $f(0) = \mu_0$ and $f(\pm ia_j) = \mu_j$. In fact, f is a function of x^2. Hence, the complex deformation is given by

$$g := W'(x)^2 + f(x) + y^2 + z^2 + w^2 = 0\,. \tag{2.23}$$

Now the exceptional $\mathbb{P}^1$'s have been replaced by the finite size S^3's. Under orientifolding, the 3-sphere S^3 located at $x = 0$ is invariant, and the 3-sphere S^3 located at $x = ia_j$ maps to one located at $x = -ia_j$ and vice-versa. Because of this, we may restrict our discussion to the singular points lying over the upper half x-plane, i.e. $x = 0$ and $x_j = ia_j$. Under the complex deformation (2.23), each of the $n + 1$ critical points in the upper half x-plane will split into two critical points denoted

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by $x = 0^+, 0^-, ia_1^+, ia_1^-, \ldots, ia_n^+, ia_n^-$. On this deformed Calabi-Yau, we can find a symplectic basis for the 3-cycles which consists of $2n+1$ compact A_i cycles and $2n+1$ noncompact B_i cycles. We will only consider the $n+1$ 3-cycles supported over upper half x-plane. As in the case of the conifold, the rational three form

$$\Omega = 2\frac{dx \wedge dy \wedge dz}{\partial g/\partial w} \tag{2.24}$$

will be a holomorphic three form on this deformed Calabi-Yau. The periods of Ω over A_j cycles supported on the upper half x-plane are given by

$$S_0 = \pm\frac{1}{2\pi i}\int_{0^-}^{0^+} \omega \qquad S_j = \pm\frac{1}{2\pi i}\int_{ia_j^-}^{ia_j^+} \omega\,, \tag{2.25}$$

where the sign depends on the orientation; the periods over the dual B_j cycles are

$$\Pi_0 = \frac{1}{2\pi i}\int_{0^+}^{\Lambda_0^{3/2}} \omega \qquad \Pi_j = \frac{1}{2\pi i}\int_{ia_j^+}^{\Lambda_0^{3/2}} \omega\,. \tag{2.26}$$

Here ω is obtained by integrating Ω over the fibers of the projection to x-coordinate as before. Thus

$$\omega = dx\ \left(W'(x)^2 + f(x)\right)^{1/2}. \tag{2.27}$$

Under such a projection, the complex deformed conifold maps into a hyperelliptic curve of the form

$$y^2 = W'(x)^2 + f(x)\,. \tag{2.28}$$

After the transition, the D-branes give rise to N_j units of H_{RR} flux through the j-th S^3 cycle A_j and α units of H_{NS} flux through each of the dual non-compact B_j cycles, with $2\pi i\alpha = 8\pi^2/g_0^2$ given in terms of the bare coupling constant g_0 of the original 4d O(N) field theory. Instead, the H_{RR} flux through the S^3 seated at the origin is reduced by the orientifolding procedure. We now have the superpotential in terms of the A_i and B_i periods as

$$-\frac{1}{2\pi i}W_{\text{eff}} = \left(\frac{N_0}{2} \mp 1\right)\Pi_0 + \sum_{j=1}^{n} N_j\Pi_j + \alpha\left(\sum_{j=0}^{n} S_j\right). \tag{2.29}$$

The above Kähler deformed Calabi-Yau is almost identical to that of a partial resolution of the orbifolded conifold under Z_n action studied in [4]. The studied partial resolution of the orbifolded conifold by a Z_n has n ordinary conifold singularities along $n-1$ $\mathbb{P}^1$'s and the distance between them is controlled by the size of $\mathbb{P}^1$ cycles. The intersection matrix of these $\mathbb{P}^1$ cycles is the same as the Cartan matrix of SU(n) group. In [4], this issue has been studied for the duality between type IIB string theory on $AdS_5 \times X_5$ and the $\mathcal{N}=1$ field theory living on the world-volume of the D3-branes placed on the orbifolded conifold singularity. Here X_5 is the horizon of the orbifolded conifold. It would be interesting to formulate a large N duality via geometric transition in this context.

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3. Field theory results and large N duality

Based on Chern-Simons/topological string duality [7], in [6] a new duality has been formulated, stating that the large N limit of the field theory obtained on N D6-branes wrapping the special lagrangian 3-cycle of a deformed conifold is dual to type IIA strings propagating on the blow-up of the conifold, the latter being a $\mathcal{O}(-1)+\mathcal{O}(-1)$ bundle over $\mathbb{P}^1$. In the dual/string picture we have N units of RR flux through the $\mathbb{P}^1$ and NS flux through the dual 4-cycle. The decoupling limit occurs when N is large and by duality the size of the blown-up sphere gets identified with the glueball chiral superfield and the expectation value of its lowest component corresponds to gaugino condensation.

By mirror symmetry, the type IIB picture is obtained, where the original $\mathrm{U}(N)$ theory takes place on the world-volume of D5-branes wrapped on the blown-up cycle of the resolved conifold, and the dual picture is type IIB on the deformed conifold background with fluxes but without branes. In presence of a generic superpotential, the number of blown-up cycles increases. The results of [17] state that the gauge group obtained after the compactification of type IIB string is $\mathrm{U}(1)^n$ were n is the number of compact 3-cycles. The $\mathcal{N}=2$ supersymmetry is broken to $\mathcal{N}=1$ by the presence of electric and magnetic fluxes [16] so that the field theory obtained in the deformed conifold side is $\mathcal{N}=1$, $\mathrm{U}(1)^n$ gauge theory.

The main claim of this paper is that the above $\mathrm{U}(1)^n$ gauge fields coincide with those of the theory $\mathrm{O}(N_0)\times\mathrm{U}(N_1)\times\cdots\times\mathrm{U}(N_n)$, that results from the low energy excitations around a generic Higgs vacuum of the $\mathrm{O}(N)$ theory with superpotential (2.15), after the $\mathrm{SO}(N_0),\mathrm{SU}(N_1),\ldots,\mathrm{SU}(N_n)$ factors get a mass gap and confine, the $Z_2=\mathrm{O}(N_0)/\,\mathrm{SO}(N_0)$ being a global group. The exact quantum effective coupling constants should coincide with the above τ_{ij}. In particular, the arguments of [11] showing that τ_{ij} is the period matrix of a hyperelliptic curve extends to our case, the curve being precisely (2.28). We do this by using the complex deformation of the geometry from the previous section and show that the coupling constants can be obtained from two identified hyperelliptic curves: one is the Seiberg-Witten curve of the field theory leaving on the world-volume of the D5-branes on the Kähler deformed conifold, and the other is connected to the complex deformation, and it is given by (2.28).

3.1 Undeformed theory

Before going into the details of the deformed model, we first discuss the pure $\mathcal{N}=1$, SO field theories. For the $\mathrm{U}(N)$ case, the large N duality was formulated in [11] in purely gauge theoretic terms. In the complex deformed side, the gluino condensation left only the $\mathrm{U}(1)$ subgroup. In the Kähler deformed side, this $\mathrm{U}(1)$ group was identified with the $\mathrm{U}(1)$ gauge theory coming from the compactification on the conifold and by turning electric and magnetic Fayet-Iliopoulos superpotential terms which softly break $\mathcal{N}=2$ to $\mathcal{N}=1$ [16, 17].

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In the presence of the orientifold, the actual situation in the complex deformed conifold side is that on the D6-branes wrapped on the 3-cycle we have the group $\mathrm{O}(N) = \mathrm{SO}(N) \times Z_2$ where the group Z_2 is a discrete one [18]. On the other hand, the orientifold does not affect the 3-cycle but implies a Z_2 projection on all the other coordinates so the above compactification on the conifold will give a Z_2 group instead of U(1). This geometrical Z_2 group is identified with the group Z_2 on the world-volume theory for the D6-branes. As in [17], there is also a charged hypermultiplet under the Z_2 group which is to be identified with the baryon field of the $\mathrm{SO}(N)$ theory as discussed in [18].

Therefore the duality can be formulated in purely gauge theoretic terms. We shall see next how things go in the presence of the superpotential.

3.2 Moduli space for the deformed theory

We now consider an $\mathcal{N}=1$ supersymmetric gauge theory with adjoint chiral superfield Φ and the tree level superpotential

$$W_{\text{tree}} = \sum_{k=1}^{n+1} \frac{g_k}{2k} \mathrm{Tr}(\Phi^{2k})\,. \tag{3.1}$$

Without the superpotential, the theory would have $\mathcal{N}=2$ supersymmetry.

The classical theory with the superpotential (3.1) has many vacua as discussed in [19]. If we rotate the field Φ into a 2×2 block form $\mathrm{diag}(x_1 i\sigma_2, \ldots, x_N i\sigma_2)$ for $\mathrm{SO}(2N)$ and $\mathrm{diag}(x_1 i\sigma_2, \ldots, x_N i\sigma_2, 0)$ for $\mathrm{SO}(2N+1)$, with σ_2 the Pauli matrix, the supersymmetric ground states are given by the zeroes of

$$W'(x) = g_{n+1}\, x \prod_{j=1}^{n} (x^2 + a_j^2) = 0\,, \tag{3.2}$$

and the ground states are labeled by a set of integers $(N_0, N_1, \ldots, N_n)$; each N_j giving the number of eigenvalues x_i of $\langle \Phi \rangle$ which are equal to 0 or ia_j. In this vacuum the gauge group is broken to

$$\mathrm{SO}(2N+1) \to \mathrm{SO}(2N_0+1) \times \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_n) \tag{3.3}$$

or

$$\mathrm{SO}(2N) \to \mathrm{SO}(2N_0) \times \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_n) \tag{3.4}$$

with $\sum_{j=0}^{n} N_j = N$. These breaking patterns were geometrically engineered in the previous section.

At low energies, for each $\mathrm{U}(N_i)$ factor in (3.3)–(3.4), the corresponding $\mathrm{SU}(N_i)$ develops a mass gap and confine, the remaining U(1) staying massless. In total, the low energy theory has then $\mathrm{U}(1)^n$ symmetry, and we will identify the corresponding couplings τ_{ij} with the second derivative of the prepotential of the complex deformed conifold.

3.3 The low energy superpotential

In this section we would like to present a novel derivation of the exact low-energy counterpart of the superpotential (3.1), which is usually claimed to be given simply by its vacuum expectation value, without further modifications to the Seiberg-Witten solution. In principle, it seems that this is the case only for small values (with respect to appropriate powers of Λ) of the parameters g_k. However, it was already argued by Seiberg and Witten in the case of deformations of SU(2) super Yang-Mills theory parametrized solely by g_1, that the result is exact for any value of g_1 [20].

The most natural framework to generalize this statement for a generic deformation $\{g_k\}$ is provided by the Whitham hierarchy associated to $\mathcal{N}=2$ supersymmetric gauge theories [21, 22, 23]. Let us proceed for simplicity in the case of SU(N). Consider the parameters g_k as components of $\mathcal{N}=2$ vector multiplets T_k,

$$T_k = t_k + \tilde{\theta}^2 g_k + \cdots\,, \tag{3.5}$$

where the dots amount to the remaining components of the superfield. The fields T_k enter into the effective prepotential almost on the same footing than the coordinates A_i of the Coulomb branch. The main difference is that they are monodromy invariants and so must be their duals T_k^D. Moreover, notice that a semiclassical contribution to the prepotential of the form

$$\delta\mathcal{F}_k = T_k\; U_{k+1}\,, \tag{3.6}$$

where U_{k+1} is the chiral superfield corresponding to $\mathrm{Tr}(\Phi^{k+1})$, leads, after integration in $\tilde{\theta}^2$, to a superpotential like term of the form

$$\delta W_{class} = g_k \frac{\partial \mathcal{F}}{\partial T_k} = g_k U_{k+1}\,. \tag{3.7}$$

The theory is nevertheless still $\mathcal{N}=2$ invariant since it is constructed out of $\mathcal{N}=2$ vector multiplets. The supersymmetry is softly broken to $\mathcal{N}=1$ if we freeze the T_k fields as[1]

$$T_1 = 1/g_0^2 + \tilde{\theta}^2 g_1 \qquad T_k = \tilde{\theta}^2 g_k \quad k>1\,, \tag{3.8}$$

g_0 being the bare coupling constant. It is immediate to see that the semiclassical contribution to the effective superpotential is (3.7), and correspond to the microscopic deformation we are interested in. In order to write down the low energy effective action involving all quantum corrections, we should be able to compute $\frac{\partial\mathcal{F}}{\partial T_k}$ exactly. This is indeed possible, provided we interpret the T_k superfields as Whitham slow times [23]. The Whitham hierarchy provides a framework in which first and second order derivatives of the prepotential with respect to the slow times can be computed [22, 23]. When these times are promoted to spurion superfields, after freezing

[1]A complete discussion of the soft breaking to $\mathcal{N}=1$ by means of the spurion mechanism in the case corresponding to a quadratic superpotential was given in [24].

their components either to softly break to $\mathcal{N}=0$ or to $\mathcal{N}=1$, only these lower order derivatives contribute to the effective action. This allows writing an exact effective potential. The case of a generic soft breaking to $\mathcal{N}=0$ was addressed in [23]. When supersymmetry is broken to $\mathcal{N}=1$, as in the present paper, the only non vanishing contribution to the effective potential is given by the first order derivative of the prepotential with respect to T_k, which is exactly[2] U_k. This result also holds in the case of SO / Sp gauge groups [25]. This shows that the exact low energy effective superpotential (besides the contribution of BPS massless states) is given by

$$W_{\mathrm{low}} = \sum_{k=1}^{n+1} g_k U_k \,. \tag{3.9}$$

In the case of $\mathrm{SO}(N)$, we should replace $U_k \to U_{2k}$.

3.4 Strong coupling dynamics

The $\mathcal{N}=2$ theory deformed by (3.1) only has unbroken supersymmetry on submanifolds of the Coulomb branch, where there are additional massless fields besides the u_r. They are nothing but the magnetic monopoles or dyons which become massless on some particular submanifolds $\langle u_p\rangle$ where the Seiberg-Witten curve degenerates. Near a point with l massless monopoles, the superpotential is

$$W = \sqrt{2}\sum_{i=1}^{l} M_i A_i \tilde{M}_i + \sum_{k=1}^{n+1} g_k U_{2k} \,, \tag{3.10}$$

where A_i denotes the chiral superfield of the U(1) vector multiplet corresponding to an $\mathcal{N}=2$ dyon hypermultiplet M_i. The vevs of the lowest components of A_i, M_i, U_{2k} are written as a_i, m_i, u_{2k}. The supersymmetric vacua are at those $\langle u_{2k}\rangle$ satisfying:

$$a_i(\langle u_{2k}\rangle) = 0\,, \qquad g_p + \sqrt{2}\sum_{i=1}^{l} \frac{\partial a_i}{\partial u_{2p}}(\langle u_{2k}\rangle) m_i \tilde{m}_i = 0\,, \tag{3.11}$$

for $p = 1,\ldots,n+1$. The value of the superpotential at this vacuum is given by

$$W_{\mathrm{eff}} = \sum_{k=1}^{n+1} g_k \langle u_{2k}\rangle \,. \tag{3.12}$$

Before orientifolding, there are N massless photons in the original $\mathcal{N}=2$, $\mathrm{U}(N)$ theory while there are $2n+1$ massless photons in the vacuum after the gauge group is broken as

$$\mathrm{U}(N) \to \mathrm{U}(N_0)\times \mathrm{U}(N_1)_- \times \mathrm{U}(N_1)_+ \times \cdots \times \mathrm{U}(N_n)_- \times \mathrm{U}(N_n)_+\,, \tag{3.13}$$

[2] The discussion here is oversimplified. The actual situation is that these derivatives give a homogeneous combination of the Casimirs of order k, but we can always reacommodate the microscopic superpotential analogously, and call g_k the corresponding coefficients.

where $N_0 + 2\sum_{i=1}^{n} N_i = N$. Here $\mathrm{U}(N_j)_-$ (resp. $\mathrm{U}(N_j)_+$) is associated with N_j D5-branes wrapping $\mathbb{P}^1$ located at $-ia_j$ (resp. $+ia_j$).

If we now act by orientifolding, the original $\mathcal{N} = 2$, SO theory contains N massless photons whereas, in the low energy theory at the vacua (3.3)–(3.4), only n photons remain massless. Thus, there are $N - n$ mutually local magnetic monopoles at the orientifold vacua (3.3)–(3.4) which become massless and get an expectation value as in (3.11), $\langle m_i \tilde{m}_i \rangle \neq 0$ for $i = 1, \ldots, N - n$. The vacuum obtained from integrating out u_{2k} as in (3.11), will give $\langle u_{2k} \rangle$ in terms of the g_k. Solving for the supersymmetric vacua as in (3.11), is equivalent to minimizing W_{low}, subject to the constraint that $\langle u_p \rangle$ lie on the codimension $N - n$ subspace of the Coulomb branch where at least $N - n$ mutually local monopoles or dyons are massless.

Let us consider, for definiteness, the case of $\mathrm{SO}(2N)$ gauge theory. The corresponding discussion for $\mathrm{SO}(2N+1)$ and $\mathrm{Sp}(N)$ follows immediately. Recall that the Seiberg-Witten curve for the $\mathrm{SO}(2N)$ theory[3] is

$$y^2 = P_{2N}(x^2; u_{2k})^2 - \Lambda^{4N-4} x^4 \,, \tag{3.14}$$

where

$$P_{2N}(x^2; u_{2k}) = \det(x - \Phi) = \prod_{i=1}^{N} (x^2 - x_i^2) = \sum_{i=0}^{N} s_{2i} x^{2(N-i)} \,, \tag{3.15}$$

The s_{2k} and u_{2k} are related each other by Newton's formula

$$2k s_{2k} + \sum_{i=1}^{k} s_{2k-2i} u_{2i} = 0 \,, \qquad k = 1, 2, \ldots, N \tag{3.16}$$

with $s_0 = 1$. This curve has Z_2 symmetry. By taking quotient of the original hyperelliptic curve (3.14) of genus $2N - 1$, we will get a hyperelliptic curve of genus N which will correctly produce the periods (and hence the field theory). The original curve (3.14) is a double covering of the curve obtained by the Z_2 quotient. In our geometric engineering point of view, the Z_2 symmetry corresponds to the orientifolding which had put the constraint on the deformation. Hence we may compare $\mathcal{N} = 1$ theory from our large N duality with $\mathcal{N} = 2$ Seiberg-Witten theory in both ways, i.e. $\mathrm{U}(2N)$ theory with orientifold symmetry with the original curve of genus $2N - 1$ with Z_2 symmetry or $\mathrm{O}(2N)$ theory obtained after orientifolding with the curve of genus N after the Z_2 quotient. But since we have arranged the adjoint Φ to have purely imaginary eigenvalues in the orientifolding, the orientifold will produce Z_2 symmetry of the curve after changing its complex structure. Therefore, the condition on the

[3]The Seiberg-Witten curve for SO / Sp theories can be written both as a hyperelliptic curve of genus r (r the rank of the group) [26] or, instead, as a hyperelliptic curve of genus $2r - 1$ with Z_2 symmetry [27]. The second choice is more naturally connected to the geometrical picture where we have the Z_2 symmetry.

original curve (3.14) for having $N-n$ mutually local massless magnetic monopoles is that, after shifting x if necessary,

$$y^2 = P_{2N}(x^2; \langle u_{2k} \rangle)^2 - \Lambda^{4N-4} x^4 = x^2 (H_{2N-2n-2}(x))^2 F_{4n+2}(x)\,, \tag{3.17}$$

where $H_{2N-2n-2}$ is a polynomial in x of degree $2N-2n-2$ and F_{4n+2} is a polynomial in x of degree $4n+2$. Actually, both H and F are polynomials in x^2.

The photons which are left massless in (3.2) have gauge couplings which are given by the period matrix of the reduced curve

$$y^2 = F_{4n+2}(x^2; \langle u_{2k} \rangle) = F_{4n+2}(x^2; g_k, \Lambda)\,, \tag{3.18}$$

and from this we can read the gauge couplings of the $\mathrm{U}(1)^n$ group in terms of g_k, Λ.

Let us briefly discuss about the Seiberg-Witten curve after the quotient to clarify its connection with the number of massless dyons at the vacuum $\langle u_{2k} \rangle$. The original curve (3.14) will become a curve of genus N while the quotient of the reduced curve (3.18) by Z_2 will be of the form

$$y^2 = x \tilde{F}_{2n+1}(x) \tag{3.19}$$

and its genus is n. Here $\tilde{F}_{2n+1}(x)$ is obtained by replacing x^2 by x in $F_{4n+2}(x^2)$. Hence $N-n$ cycles has shrunken and these correspond to massless magnetic monopoles.

Recall from the previous section that the dual Calabi-Yau geometry is determined by

$$W'(x)^2 + f(x) + y^2 + z^2 + w^2 = 0\,, \tag{3.20}$$

this determining the coupling constants τ_{ij},

$$\tau_{ij} = \frac{\partial \Pi_i}{\partial S_j}\,, \tag{3.21}$$

that, as shown in [11] for the SU case, can be identified with the period matrix of the hyperelliptic curve (2.28). Thus, in order to show that the τ_{ij} from (3.18) agrees with that obtained from the dual geometry, we need to prove that $F_{4n+2}(x^2)$ is related to the superpotential by

$$g_{n+1}^2 F_{4n+2}(x) = W'(x)^2 + f(x)\,. \tag{3.22}$$

Here the factor g_{n+1}^2 is inserted to match the coefficient of the highest term in both sides. This identity would provide very strong evidence for the proposed duality. It is a non-trivial statement which is difficult to prove in general. In the SU case, for example, it was shown only for the case of cubic superpotentials. We will prove it up to the coefficients of $f(x)$, that is, for the terms of the polynomials of degree greater than $2n$.

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To prove the statement, we denote the roots of $H_{2(l-1)}$ and $F_{4N-4l+2}$ of the curve (3.17) with l massless dyons by $\pm p_i$ and $\pm q_i$ and so we have

$$H_{2(l-1)}(x) = \prod_{i=1}^{l-1}(x^2 - p_i^2)\,, \qquad F_{4N-4l+2}(x) = \prod_{i=1}^{2N-2l+1}(x^2 - q_i^2)\,. \tag{3.23}$$

We may assume that p_i and q_i are non-zero numbers because x is factored out in (3.17)[4]. Then, there is a relation between the parameters g_k and the dyon vevs m_i^2 [13]

$$g_k = \sum_{i=1}^{l-1}\sum_{j=0}^{N} s_{2(j-k)} p_i^{2(N-j)} \omega_i\,, \tag{3.24}$$

where

$$\omega_i = \frac{m_{i,dy}^2}{2\prod_{s\neq i}^{l}(p_i^2 - p_s^2)\prod_{t\neq i}^{2N-2l}(p_i^2 - q_t^2)^{1/2}}\,. \tag{3.25}$$

Following the arguments of [13], we obtain a convenient form for the superpotential

$$\begin{aligned} W'_{cl}(x) &= \sum_{k=1}^{N} g_k x^{2k-1} \\ &= \sum_{k=1}^{N}\sum_{i=1}^{l-1}\sum_{j=0}^{N} x^{2k-1} s_{2(j-k)} p_i^{2(N-j)} \omega_i \\ &= \sum_{k=-\infty}^{N}\sum_{i=1}^{l-1}\sum_{j=0}^{N} x^{2k-1} s_{2(j-k)} p_i^{2(N-j)} \omega_i \pm \Lambda^{2N-2}\sum_{i=1}^{l} p_i^2\omega_i x^{-1} + \mathcal{O}(x^{-2}) \\ &= \sum_{i=1}^{l-1}\frac{P_{2N}(x^2;\langle u_{2k}\rangle)}{x(x^2-p_i^2)}\,\omega_i \pm \Lambda^{2N-2}\sum_{i=1}^{l} p_i^2\omega_i x^{-1} + \mathcal{O}(x^{-2})\,. \end{aligned} \tag{3.26}$$

We define a polynomial B_{2l-4} of degree of $2l-4$ by

$$\sum_{i=1}^{l-1}\frac{\omega_i}{x(x^2-p_i^2)} = \frac{B_{2l-4}(x)}{xH_{2(l-1)}(x)}\,, \tag{3.27}$$

with $H_{2(l-1)}(x)$ as in (3.23). Then we have

$$W'_{cl}(x) + \omega\Lambda^{2N-2}x^{-1} = B_{2l-4}(x)\sqrt{F_{4N-4l+2}(x) + \frac{\Lambda^{4N-4}x^2}{H_{2(l-1)}(x)^2}} + \mathcal{O}(x^{-2})\,, \tag{3.28}$$

where $\omega = \mp\sum_{i=1}^{l} p_i^2\omega_i$. Since the highest order in $W'_{cl}(x)$ is $g_{n+1}x^{2n+1}$, we see that $B_{2l-4}(x)$ should be of order $2n - 2N + 2l$. This shows that $l \geq N - n$ and for $l = N - n$, $B_{2l-4}(x)$ becomes a constant equal to g_{n+1}. By squaring (3.28), we obtain

$$g_{n+1}^2 F_{4n+2}(x) = W'(x)^2 + 2g_{n+1}\omega\Lambda^{2N-2}x^{2n} + \mathcal{O}(x^{2n-2})\,. \tag{3.29}$$

[4] This assumption was not spelled out explicitly in [13, (2.17)].

We have thus showed $g_{n+1}^2 F_{4n+2} = W'^2 + f(x)$ (3.22), and

$$f(x) = 2g_{n+1}\omega\Lambda^{2N-2}x^{2n} + \mathcal{O}(x^{2n-2})\,. \tag{3.30}$$

To show that both sides of (3.22) are exactly the same, we would need to consider lower order terms which depend nontrivially on N_i. It is highly desirable to obtain this result exactly, which would imply that the exact value of $\tau_{ij}(g_r,\Lambda)$, corresponding to the $\mathrm{U}(1)^n$ massless photons, found using the reduced $\mathcal{N}=2$ curve (3.18) evaluated in the $\mathcal{N}=1$ supersymmetric vacua, is consistent with that of (3.21), found via the large N duality from the proposed geometric transition.

The result for Sp is obtained in a similar way and we will not give the details because they are analogous to the ones presented above. The deformation will be of an identical form, this time the adjoint field Φ being a symmetric 2-tensor and determines a gauge symmetry breaking:

$$\mathrm{Sp}(2N) \to \mathrm{Sp}(2N_0) \times \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_n)\,. \tag{3.31}$$

In order to check the consistency, one compares the periods obtained from the geometry which are stated in the section 2 and the ones given by the reduced Seiberg-Witten curve discussed in [28, 29]. Finally, let us remark that another important check of this duality for SO / Sp gauge theories will be provided by a detailed computation of the superpotential on both dual theories.

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